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Optimizing Fuzzy Nonlinear Programming Problems Through Effective Ranking Methods

Bharathi Dharmaraj[1], Saraswathi Appasamy[1,*], Sanaz Hami Hassan Kiyadeh[2]

[1] Department of Mathematics, College of Engineering and Technology, SRM Institute of Science and Technology, Kattankulathur 603203, India; bd5402@srmist.edu.in; saraswaa@srmist.edu.in.

[2] Department of Mathematics, The University of Alabama, Alabama, USA; shkiyadeh@crimson.ua.edu.

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Abstract

This research addresses the challenges of Fuzzy Nonlinear Programming Problems (FNPPs), especially those with decision parameters defined by triangular fuzzy numbers. The methodology involves initially converting these fuzzy numbers into precise values through a robust ranking technique, improving subsequent analyses' clarity and practicality. Following this conversion, a crisp nonlinear programming problem is formulated to enable a more straightforward and deterministic approach. The Kuhn-Tucker conditions are then applied to systematically explore and analyze the problem space, identifying an optimal solution. This study delves into the intricacies of fuzzy nonlinear programming, integrating robust ranking and Kuhn-Tucker optimization to effectively solve complex issues in this domain.

Keywords: Fuzzy nonlinear programming problem, Ranking method, Triangular fuzzy number, Kuhn Tucker Condition.

1 | Introduction

Throughout the span of several years, conventional optimization techniques have played a crucial role in effectively addressing challenges associated with decision-making. The decisions formulated by individuals are grounded in scientific principles, relying on information derived from mathematical modeling environments. Despite the demonstrated success of these techniques, their real-world application often encounters impediments due to inherent ambiguities and inexactness associated with decision parameters. These complexities emanate from various sources, including errors in estimation, information gaps, imperfect representation of data, subjectivity, and reliance on human opinions. All in attempting to grapple with these

Corresponding Author: saraswaa@srmist.edu.in
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real-world challenges, conventional mathematical techniques face limitations. The inadequacies in addressing uncertainties and inaccuracies necessitate the exploration of alternative approaches. In this context, fuzzy optimization techniques emerge as a valuable and potent tool for optimization within a fuzzy environment. Fuzzy optimization methods provide a nuanced and adaptive framework that better accommodates the inherent complexities of decision-making processes affected by imprecision and uncertainty. This research delves into the significance of incorporating fuzzy optimization techniques as a robust solution to enhance the formulation and successful resolution of real-world optimization problems.

The researchers [1] employed the robust ranking method to tackle fuzzy transportation problems. The researcher [2] presented a case study involving allocating subjects within an educational institution, utilizing the robust ranking method. This methodology aimed to enhance the precision and applicability of the solution to the assignment problem. This application implies the utilization of robust ranking for optimizing subject allocation, potentially ensuring a fair and efficient distribution based on predefined criteria [3]. This novel technique introduces innovative methods for handling quadratic problems with uncertainties, offering a fresh perspective on fuzzy neutrosophic optimization. The researchers [4] proposed using a robust ranking technique to optimize fuzzy assignment models.

The researchers [5] applied the robust ranking method to solve problems involving fuzzy octagonal numbers. The researcher [6] applied robust ranking in the realm of two-phase fuzzy optimization. The researchers [7] proposed a novel method for solving Fuzzy Linear Programming (FLP) problems based on the fuzzy linear complementary problem. The FLCP provides a theoretical foundation for addressing the duality and optimality conditions in FLP, thereby facilitating the development of efficient solution techniques. The researchers proposed a solution approach tailored specifically for FFQP problems. By leveraging mathematical techniques and optimization algorithms, the authors aim to provide an effective method for finding optimal or near-optimal solutions to FFQP instances. This contribution likely provided valuable insights into fuzzy set theory's broader implications and potential applications across diverse fields, thereby solidifying its significance and applicability. It also conducted a study focused on Hungarian assignment problems incorporating fuzzy numbers. They employed robust ranking techniques to convert the fuzzy assignment problem into a crisp formulation [8].

Applying robust ranking techniques likely played a crucial role in effectively resolving optimization problems characterized by imprecise and uncertain information across two distinct phases. This exploration likely contributes valuable insights into the theoretical aspects of optimizing nonlinear fuzzy problems, advancing the understanding of optimal solutions. The researchers proposed an innovative approach for addressing fuzzy neutrosophic quadratic problems. The researchers explored the diverse applications of linear and nonlinear programming in industry. They provide an in-depth analysis of how these optimization techniques are employed to address complex challenges and enhance operational efficiency across different domains. The paper covers theoretical foundations, algorithmic approaches, and practical case studies showcasing the effectiveness of LP and NLP in industrial settings [9].

The researchers [10] introduced a methodology that transforms a fuzzy problem into a parametric form. By subsequently applying Kuhn-Tucker conditions, they derived the optimal solution for the problem [10]. The researchers' approach offers a structured way to address the inherent fuzziness in problem formulations, facilitating the attainment of optimized solutions. Utilizing robust ranking implies a strategic approach to managing uncertainties in transportation problems, potentially leading to more resilient and reliable solutions [11]. Zadeh [12] introduced the Fuzzy set concept in 1965, a seminal contribution that addressed real-world challenges. This expansion involved the integration of fuzzy logic into decision-making processes, acknowledging and accommodating uncertainties within the decision-making framework. In 1991, Zimmermann [13] furthered the understanding and application of Fuzzy set theory by delving into its theoretical foundations and practical uses.

The paper's main contribution is likely to propose a nonlinear approach to solving nonlinear programming problems. The author may introduce a novel mathematical formulation or algorithm that utilizes nonlinear

techniques to address the complexities introduced by neutrosophic parameters in linear programming problems. This approach may involve nonlinear optimization methods or transformations to convert nonlinear programming problems into equivalent forms amenable to solutions [14]. The researchers [15] provided a comprehensive overview of nonlinear models proposed for fully FLP, encompassing various formulations and solution techniques. The authors categorize existing methodologies based on their mathematical structures, highlighting the evolution of fully FLP models from linear to nonlinear forms. The survey discusses the advantages and limitations of different approaches and identifies avenues for future research in enhancing the computational efficiency and applicability of nonlinear fully FLP models [15].

The researchers' development of a new ranking function for triangular neutrosophic numbers and its application in integer programming represents a significant advancement in decision science and optimization. Their contributions offer enhanced techniques for handling uncertainty and indeterminacy in decision-making processes, with implications for a wide range of applications in areas such as logistics, finance, and engineering. Further research may explore extensions of their work, including integrating the new ranking function into other optimization frameworks and developing algorithms for efficient computation in large-scale problems [16]. The researchers [17] provide an overview of separable Fuzzy Nonlinear Programming Problems (FNPPs) and existing optimization techniques for addressing them. The authors discuss the challenges posed by fuzzy parameters in optimization and the importance of decomposing the objective function and constraints into separable components. They review traditional optimization methods and highlight the need for innovative approaches to solve separable FNPPs efficiently [17].

The researchers will demonstrate how pivotal operations on triangular fuzzy numbers can effectively solve FNPPs [18]. This could include theoretical discussions, mathematical formulations, and illustrative examples to showcase the applicability and advantages of the proposed approach. The researchers proposed that Fuzzy Linear Fractional Programming (FLFP) extends traditional linear fractional programming to handle fuzzy objectives and constraints, offering a flexible approach to decision-making under uncertainty. This paper presents a fuzzy mathematical approach for solving multiobjective FLFPs using triangular to trapezoidal fuzzy numbers, aiming to provide decision-makers with robust solutions that account for both imprecision and multiple conflicting objectives [19].

The researcher [20] focused on solving FLP problems in multidimensional spaces, proposing new methodologies for handling fuzzy constraints and objectives using advanced computational techniques. The researcher [21] introduces a novel algorithm for linear programming that eliminates the need for artificial variables, streamlining the solution process and enhancing computational efficiency. The researchers [22] propose a new method for addressing multiobjective linear programming problems using triangular neutrosophic numbers, integrating theoretical foundations and practical applications. The researcher [23] discusses the application of linear fractional programming with fuzzy parameters in the industrial sector, providing case studies and practical insights. The researchers [24] explore LR-type Pythagorean FLP problems, presenting solutions under equality constraints and discussing their implications in fuzzy optimization. The researchers develop an extended Data Envelopment Analysis (DEA) method to address multiobjective transportation problems using Fermatean fuzzy sets. Using the primal simplex algorithm, the researcher addresses linear programming problems with cost coefficients and grey resources, offering solutions relevant to big data contexts. The researchers [25] aim to characterize solutions for complex programming problems with fuzzy constraints, providing a comprehensive framework and examples.

The researchers [26] proposed a hybrid Particle Swarm Optimization (PSO) algorithm to tackle single-machine scheduling issues, accounting for sequence-dependent setup times and learning effects. The researcher [27] presents a nonlinear programming method to determine preference weights from incomplete comparisons, enhancing decision-making processes in industrial engineering. The researcher [28] discusses optimization techniques for FLFP problems using fuzzy numbers, focusing on big data applications. The researchers [29] introduce a neutrosophic goal programming approach for multiobjective linear fractional programming problems, addressing uncertainties in decision-making.

The researchers [30] proposed a linearization technique for multiobjective linear plus linear fractional programming, enhancing solution accuracy and efficiency, and they examined service pattern modifications in the context of social distancing and uncertainty, proposing operational research strategies to adapt to these conditions. The researchers [31] develop a multiobjective programming approach for solving integer-valued neutrosophic shortest path problems, enhancing decision-making in uncertain environments. The researchers propose a new method for solving shortest-path problems using Gaussian-valued neutrosophic numbers, improving accuracy and reliability [32]. The researchers [33] introduce an interactive compromise programming approach for vendor selection problems under fuzziness, offering a robust decision-making framework.

The researchers [34] present a method for solving vendor selection problems using interval approximation of piecewise quadratic fuzzy number enhancing selection processes. The researchers [35] propose a null set concept to address fuzzy nonlinear optimization problems, offering a novel approach to handle fuzziness in optimization. The researchers [36] analyze the robust optimality of non-degenerate basic feasible solutions in linear programming with fuzzy objective coefficients, enhancing decision-making under uncertainty. The researchers [37] propose a fuzzy preference programming and weighted influence nonlinear gauge system for assessing mission architecture at NASA, integrating soft computing techniques for better evaluations. The researchers [38] solved the time-dependent shortest path problem under the bipolar neutrosophic fuzzy arc value. The researchers [39] proposed to solve the neutrosophic linear fractional programming problem with triangular neutrosophic numbers. The researcher [40] developed a fuzzy-trapezoidal DEMATEL approach method for solving decision-making problems under uncertainty.

This application underscores the versatility of robust ranking in handling various types of fuzzy numbers and optimizing solutions across diverse contexts. In this research paper, we focus on resolving FNPPs characterized by the exclusive use of triangular fuzzy numbers as decision parameters. Our methodology commences with applying the robust ranking approach to each triangular fuzzy number, systematically converting them into precise and well-defined crisp values. This fundamental step is a pivotal transformation, reforming the entire problem into a crisp nonlinear programming framework. Following the crisp representation's attainment, we formulate and address the resulting crisp nonlinear programming problem. Utilizing the Kuhn-Tucker conditions, we systematically and comprehensively explore the problem space to derive an optimal solution. This intricate process entails thoroughly examining constraints and objectives under the influence of the Kuhn-Tucker conditions, ultimately identifying the most favorable solution for the given FNPPs.

The robustness of the robust ranking method refers to its ability to consistently and reliably convert fuzzy numbers into precise values despite variations and uncertainties in the input data. This method typically involves resilient techniques against fluctuations in the fuzzy number parameters, ensuring stable and accurate transformations. By proving the robustness of the robust ranking method, researchers demonstrate its effectiveness in providing dependable results across different scenarios and datasets, enhancing its applicability in practical problem-solving contexts.

The integration of the robust ranking approach with the Kuhn-Tucker conditions signifies a robust and comprehensive methodology designed to effectively manage the complexities introduced by the inherent fuzziness in the decision parameters. Through this multi-step process, we aim to augment the precision and efficiency of solving FNPPs, offering valuable insights into the intricate interplay between robust ranking and Kuhn-Tucker optimization strategies. The subsequent portion of this paper unfolds as follows: Section two elucidates the concept of fuzzy sets along with pertinent preliminaries, while section three delineates the general form of FNPPs and explores the associated Kuhn-Tucker conditions. Section four delves into a numerical example for further illustration. The conclusive insights drawn from our discussions are presented in section five, summarizing some key conclusions at the end of this paper.

2 | Preliminaries

Definition 1. If X is a universal set and $x \in X$, then a fuzzy set $\widetilde{A}$ defined as a collection of ordered pairs.

$$\widetilde{A} = \{(x, \mu_{\widetilde{A}}(x)) | x \epsilon X\},$$

where $\mu_{\widetilde{A}}(x)$ is called the membership function that maps X to the membership space M".

Definition 2. A fuzzy number $\widetilde{A}$ on R is said to be a triangular fuzzy number if its membership function $\widetilde{A}: R \to [0, 1]$ has the following characteristics:

$$\widetilde{A}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, & a_1 \leq x \leq a_2, \\ \dfrac{a_3 - x}{a_3 - a_2}, & a_2 < x \leq a_3, \\ 0, & \text{otherwise.} \end{cases}$$

We denote this triangular fuzzy number $\widetilde{A} = (a_1, a_2, a_3)$.

Definition 3. A triangular fuzzy number $\widetilde{A} = (a_1, a_2, a_3)$ is said to be non-negative iff $a_1 \geq 0$.

Definition 4. A fuzzy set $\widetilde{A}$ defined on X is called a normal fuzzy set if there exists at least one $x \in X$, such that $\mu_{\widetilde{A}}(x) = 1$.

Definition 5. Given a fuzzy set $\widetilde{A}$ defined on X and any $\alpha \in [0,1]$ the α–cut is denoted by $\widetilde{A}(\alpha)$ and is defined as

$$\widetilde{A}(\alpha) = \{x, \mu(x) \geq \alpha, \alpha \in [0,1]\}.$$

Definition 6. If ã is a fuzzy number, the Robust ranking index is defined by

$$R(\tilde{a}) = 0.5 \int_0^1 (a_\alpha^L, a_\alpha^U)\, d\alpha,$$

where

$$(a_\alpha^L, a_\alpha^U) = \{(b - a)\alpha + a, c - (c - b)\alpha\}.$$

Is the α–cut of the fuzzy number ã.

Theorem 1 (Kuhn-Tucker conditions for nonlinear programming problem). Consider a nonlinear programming problem represented as

$$\text{Maximize (or)Minimize } f(x),$$

$$\text{s.t.}$$

$$g_j(x_j) \leq \text{or} = \text{or} \geq b_i, \quad i = 1,2,3, \ldots, m,$$

$$x_j \geq 0, \quad \text{for } j = 1,2,3, \ldots, n,$$

where $f(x)$ and $g_i(x)$ are objective and constraint functions, and x denotes decision variables.

Let μ_j be the Lagrange multiplier associated with the j^{th} constraint. The Kuhn-Tucker conditions for this optimization problem are stated as follows:

Gradient of the objective function

$$\nabla f(x) - \sum_{j=1}^{n} \mu_j \nabla g_j(x) = 0.$$

Complementary slackness conditions

$$\mu_j \nabla g_j(x) = 0, \qquad \text{for all } j = 1,2,3,\ldots,n.$$

Constraint satisfaction

$$g_j(x) - b_i \leq 0, \quad \text{for all } j = 1,2,3,\ldots,n.$$

Nonnegative of Lagrange multipliers

$$\mu_j \geq 0, \qquad \text{for all } j = 1,2,3,\ldots,n.$$

Proof: Let x^* be a feasible solution that satisfies the Kuhn-Tucker conditions. We will show that x^* is an optimal solution.

Gradient of the objective function

By the first Kuhn-Tucker condition, we have

$$\nabla f(x^*) - \sum_{j=1}^{n} \mu_j \nabla g_j(x^*) = 0.$$

This condition ensures that the objective function gradient is aligned with the gradients of the constraint functions at the optimal solution.

Complementary slackness conditions

The condition $\mu_j \nabla g_j(x^*) = 0$ implies that either the Lagrange multiplier or the gradient of the corresponding constraint function is zero. This reflects the complementary slackness condition.

Constraint satisfaction

The condition $g_j(x^*) - b_i \leq 0$ ensures the constraints are satisfied with appropriate slackness at the optimal solution.

Nonnegativity of Lagrange multipliers

The condition $\mu_j \geq 0$ guarantees that the Lagrange multiplier associated with each constraint is non-negative. Therefore, if a solution x^*satisfies all these conditions, it is an optimal solution to the given nonlinear programming problem, demonstrating the optimality of the Kuhn-Tucker conditions.

3 | Robust Ranking Method

The robust for robust ranking method is a decision-making technique used to handle uncertainties in Multi-Criteria Decision-Making (MCDM) problems. This method emphasizes robustness, meaning it seeks to find solutions that perform well across a range of possible scenarios rather than optimizing for a single, potentially uncertain scenario. This technique is employed to convert triangular fuzzy numbers into precise, crisp values. The robust ranking method is crucial for transforming fuzzy parameters into a format effectively utilized in optimization processes. This transformation allows for reforming fuzzy problems into crisp nonlinear programming frameworks. The result of applying the robust ranking method is a well-defined, crisp, nonlinear programming problem. This outcome allows the application of conventional optimization techniques to solve the problem effectively. The robust ranking method has been applied in various fields, including educational subject allocation and optimization of fuzzy assignment models. By utilizing this method, researchers aim to enhance the effectiveness and robustness of decision-making processes in practical applications.

3.1 | Robust Decision Making

This involves evaluating the performance of different alternatives under various possible future states or scenarios. The goal is to identify effective decisions despite uncertain parameters or future conditions.

3.2 | Compared with Centroid Ranking Method and Robust Ranking Method

Simplicity vs. complexity

Centroid ranking is straightforward to apply, whereas robust ranking is more intricate but offers a more detailed comparison.

Consideration of spread

Centroid ranking focuses solely on the central point of the fuzzy number, disregarding its spread or shape. In contrast, robust ranking considers both the spread and the shape of the fuzzy number.

Applicability

Centroid ranking is generally adequate for fuzzy numbers that are simple and symmetrical. However, robust ranking is more suitable for handling fuzzy numbers that are complex, asymmetric, or involve significant uncertainty.

In scenarios such as decision-making under uncertainty or optimization involving fuzzy numbers, the choice between centroid ranking and robust ranking depends on the specific characteristics of the fuzzy numbers and the desired level of detail in the ranking process.

4 | Fuzzy Nonlinear Programming Problems

The FNPP is characterized by

$$\begin{aligned} &\text{Maximize (or) Minimize } \tilde{f}(x) = \tilde{f}(x_1, x_2, \dots, x_n), \\ &\text{s.t.} \\ &\tilde{g}_j(\tilde{x}_j) \le \text{ or } = \text{ or } \ge \tilde{b}_i, \quad i = 1,2,3,\dots,m, \\ &\tilde{x}_j \ge \tilde{0}, \quad \text{for } j = 1,2,3,\dots,n. \end{aligned} \tag{1}$$

4.1 | Kuhn-Tucker Condition

The paper presents the Kuhn-Tucker conditions in their fuzzy form, which are used for optimization. These conditions are fundamental in identifying optimal solutions within the context of nonlinear programming, especially when dealing with fuzzy parameters. Integrating these conditions with the robust ranking method enhances the overall methodology. The Kuhn-Tucker conditions in their fuzzy form are presented as follows:

$$\nabla f(x) - \sum_{j=1}^{n} \mu_j \nabla g_j(x) = 0. \tag{2}$$

$$\mu_j \nabla g_j(x) = 0, \quad \text{for all } j = 1,2,3,\dots,n. \tag{3}$$

$$g_j(x) - b_i \le 0, \quad \text{for all } j = 1,2,3,\dots,n. \tag{4}$$

$$\mu_j \ge 0, \quad \text{for all } j = 1,2,3,\dots,n. \tag{5}$$

The paper innovatively combines robust ranking methods with Kuhn-Tucker optimization conditions. This integration provides a fresh perspective on resolving FNPPs, addressing the complexities associated with these types of problems.

5 | Numerical Examples

Example 1. The fuzzy objective function and constraints are given as follows:

$$\begin{aligned}
&\text{Max}\ \tilde{Z} = (1,3,4)\tilde{x}_1^2 + (1,2,3)\tilde{x}_2^2,\\
&\text{s.t.}\\
&(0,1,3)\tilde{x}_1 + (2,3,5)\,\tilde{x}_2 \leq (3,4,6),\\
&(1,2,4)\,\tilde{x}_1 - (0,1,2)\,\tilde{x}_2 \leq (1,2,5),\\
&\tilde{x}_1, \tilde{x}_2 \geq 0.
\end{aligned} \tag{6}$$

The robust ranking method is applied to transform fuzzy parameters into crisp values. The method involves the following steps:

$$R(1,3,4) = 0.5 \int_0^1 \{(3-1)\,\alpha + 1 + 4 - (4-3)\alpha\}d\alpha$$

$$= 0.5 \int_0^1 \{2\,\alpha + 1 + 4 - \alpha\}d\alpha$$

$$= 0.5 \int_0^1 (5+\alpha)\,d\alpha,$$

$$R(1,3,4) = 2.75.$$

Repeat the above process for all fuzzy coefficients to obtain crisp values:

$R(1,2,3) = 2, R(0,1,3) = 1.25, R(2,3,5) = 3.25, R(3,4,6) = 4.25, R(1,2,4) = 2.25\,, R(0,1,2) = 1, R(1,2,5) = 2.5$”.

The result is a well-defined, crisp, nonlinear programming problem. Substitute the crisp coefficients back into the objective function and constraints.

$$\text{Max}\ \tilde{Z} = 2.75\ \tilde{x}_1^2 + 2\ \tilde{x}_2^2,$$

$$\text{s.t.}$$

$$1.25\ \tilde{x}_1 + 3.25\ \tilde{x}_2 \leq 4.25,$$

$$2.25\,\tilde{x}_1 - \tilde{x}_2 \leq 2.5,$$

$$\tilde{x}_1, \tilde{x}_2 \geq 0.$$

Introduce slack variables ρ_1 and ρ_2 to handle the inequality constraints. By applying *Eq. (2)*, we get

$$5.5\,\tilde{x}_1 - 1.25\,\rho_1 - 2.25\ \rho_2 = 0. \tag{7}$$

$$4\tilde{x}_2 - 3.25\,\rho_1 + \rho_2 = 0. \tag{8}$$

By applying *Eq. (3)*, we get

$$\rho_1\,[\,1.25\ \tilde{x}_1 + 3.25\ \tilde{x}_2 - 4.25] = 0, \tag{9}$$

$$\rho_2[\,2.25\,\tilde{x}_1 - 1.00\ \tilde{x}_2 - 2.5\,] = 0. \tag{10}$$

By applying *Eq. (4)*, we get

$$\begin{aligned}
&1.25\ \tilde{x}_1 + 3.25\ \tilde{x}_2 - 4.25 \leq 0,\\
&2.25\,\tilde{x}_2 - 1.00\ \tilde{x}_2 - 2.5 \leq 0
\end{aligned} \tag{11}$$

Case 1. Let $\rho_1 = 0, \rho_2 = 0$ apply this value in *Eqs. (7)* and *(8)*, we get $\tilde{x}_1 = 0$ and $\tilde{x}_2 = 0$ (Neglete).

Case 2. Let $\rho_1 \neq 0, \rho_2 = 0$ apply this value in *Eqs. (7)-(9)*, we get

$1.25\ \tilde{x}_1 + 3.25\ \tilde{x}_2 - 4.25 = 0,$

$5.5\,\tilde{x}_1 - 1.25\,\rho_1 = 0,$

$5.5\,\tilde{x}_1 = 1.25\,\rho_1,$

$\tilde{x}_1 = \rho_1 \frac{5}{22},$

$4\tilde{x}_2 - 3.25\,\rho_1 = 0,$

$4\tilde{x}_2 = 3.25\,\rho_1\,,$

$\tilde{x}_2 = \rho_1 \frac{13}{16}.$

Substituting this value in *Eq. (9)*, we get

$(1.25)\,\rho_1 \frac{5}{22} + (3.25)\,\rho_1 \frac{13}{16} - 4.25 = 0,$

$0.2841\rho_1 + \rho_1 2.6406 = 4.25,$

$\rho_1 = 1.453,$

$\tilde{x}_1 = 0.3302\ \&\ \tilde{x}_2 = 1.1806.$

$\tilde{x}_1 \& \tilde{x}_2$ satisfies in *Eq. (11)*. So this is the optimum solution. Solving the above equations:

$\text{Max}\ \tilde{Z} = 2.75\ \tilde{x}_1^2 + 2\,\tilde{x}_2^2$

$= 2.75(0.3302\,)^2 + 2(1.1806)^2,$

$\text{Max}\ \tilde{Z} = 3.0874,$

Comparison 1. However, according to reference [10], the optimal value is (372/49,0,0). For optimality comparison, the value is converted into its crisp equivalent. Subsequently, we obtain

$\text{R}\left(\frac{372}{49}, 0{,}0\right) = 1.8969.$

Comparison *Fig. 1*.

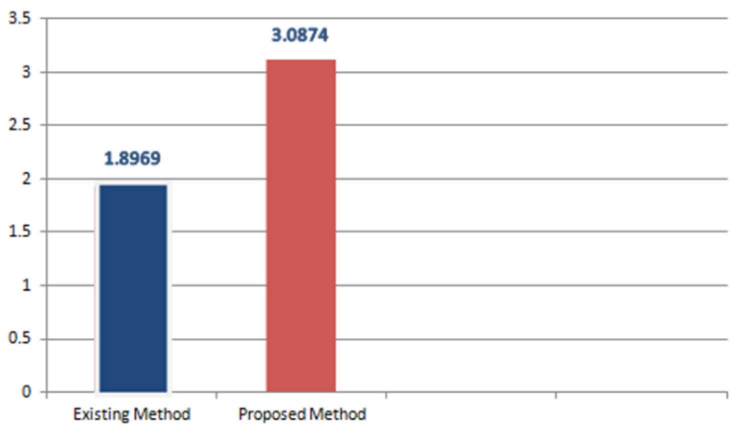

Fig. 1. Graphical representation for comparison of the proposed method.

Hence, upon comparing both solutions, it is evident that our solution is optimal.

Example 2. Examine a nonlinear programming problem under fuzzy conditions by

$$\begin{aligned}
&\text{Max } \tilde{Z} = \tilde{x}_1^2 + \tilde{x}_2^2,\\
&\text{s.t.}\\
&(0,1,2)\tilde{x}_1 + (1,2,3)\,\tilde{x}_2 \le (1,10,27),\\
&(1,2,3)\,\tilde{x}_1 + (0,1,2)\,\tilde{x}_2 \le (2,11,28),\\
&\tilde{x}_1, \tilde{x}_2 \ge 0,
\end{aligned} \tag{12}$$

The robust ranking method is applied to transform fuzzy parameters into crisp values. The method involves the following steps:

$$R(0,1,2) = 0.5\int_0^1 \{(1-0)\,\alpha + 0 + 2 - (2-1)\alpha\,\}d\alpha,$$

$$= 0.5\int_0^1 \{(\,\alpha) + 2 - (\alpha)\}d\alpha,$$

$$= 0.5\int_0^1 (2\,)d\alpha,$$

$$R(0,1,2) = 1.$$

Repeat the above process for all fuzzy coefficients to obtain crisp values.

R(0,1,2)=1,"R(1,2,3) =2, R(1,10,27)=12, R(1,2,3)=2, R(0,1,2)=1, R(2,11,28)=13.

The result is a well-defined, crisp, nonlinear programming problem. Substitute the crisp coefficients back into the objective function and constraints.

$$\begin{aligned}
&\text{Max } \tilde{Z} = \tilde{x}_1^2 + \tilde{x}_2^2,\\
&\text{s.t.}\\
&\tilde{x}_1 + 2\,\tilde{x}_2 \le 12,\\
&2\tilde{x}_1 + \tilde{x}_2 \le 13,\\
&\tilde{x}_1, \tilde{x}_2 \ge 0.
\end{aligned}$$

Introduce slack variables ρ_1 and ρ_2 to handle the inequality constraints, By applying *Eq. (2)*, we get

$$2\,\tilde{x}_1 - \rho_1 - 2\,\rho_2 = 0. \tag{13}$$

$$2\tilde{x}_2 - 2\rho_1 - \rho_2 = 0. \tag{14}$$

By applying *Eq. (3)*, we get,

$$\rho_1[\,\tilde{x}_1 + \ 2\tilde{x}_2 - 12] = 0. \tag{15}$$

$$\rho_2[2\,\tilde{x}_1 + \tilde{x}_2 - 13] = 0. \tag{16}$$

By applying *Eq. (4)*, we get,

$$\begin{aligned}
&\tilde{x}_1 + 2\,\tilde{x}_2 - 12 \le 0.\\
&2\tilde{x}_1 + \tilde{x}_2 - 13 \le 0.
\end{aligned} \tag{17}$$

Case 3. Let $\rho_1 = 0, \rho_2 = 0$ apply this value in *Eqs. (13)* and *(14)* we get, $\tilde{x}_1 = 0$ and $\tilde{x}_2 = 0$ (Neglete).

Case 4. Let $\rho_1 \ne 0, \rho_2 = 0$ apply this value in *Eqs. (13)-(15)*, we get,

$\tilde{x}_1 + 2\tilde{x}_2 - 12 = 0,$

$2\,\tilde{x}_1 - \rho_1 = 0,$

$\tilde{x}_1 = \frac{\rho_1}{2},$

$2\tilde{x}_2 - 2\rho_1 = 0,$

$\tilde{x}_2 = \rho_1,$

$\frac{\rho_1}{2} + 2\,\rho_1 - 12 \leq 0,$

$\rho_1 = 4.8,$

$\tilde{x}_1 = 2.4\,, \qquad \tilde{x}_2 = 4.8,$

$\text{Max}\,\tilde{Z} = 14.4.$

Case 5. Let $\rho_1 = 0, \rho_2 \neq 0$ apply this value in *Eqs. (13)*, *(14)* and *(16)*, we get,

$2\,\tilde{x}_1 + \tilde{x}_2 - 13 = 0,$

$2\,\tilde{x}_1 - 2\,\rho_2 = 0,$

$\tilde{x}_1 = \rho_2,$

$2\tilde{x}_2 - \rho_2 = 0,$

$\tilde{x}_2 = \frac{\rho_2}{2},$

$$2\,\rho_2 + \frac{\rho_2}{2} - 13 = 0,$$

$\rho_2 = 5.2,$

$\tilde{x}_1 = 5.2\ \&\ \tilde{x}_2 = 2.6,$

$\text{Max}\,\tilde{Z} = 15.6.$

Comparison 2. However, according to reference [10], the optimal value is (25,0,0). For optimality comparison, the value is converted into its crisp equivalent. Subsequently, we obtain,

$\text{R}\,(25{,}0{,}0) = 6.2.$

Comparison *Fig. 2*.

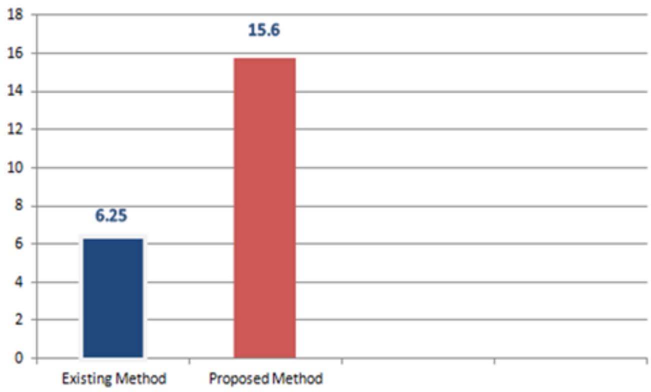

Fig. 2. Graphical representation for comparison of the proposed method.

Hence, upon comparing both solutions, it is evident that our solution is optimal.

6 | Limitations

This research introduces a comprehensive and innovative approach to tackling FNPPs with triangular fuzzy numbers. Despite its merits, the proposed methodology has certain limitations that warrant acknowledgment:

Limited generalizability

The methodology is specifically crafted for FNPPs featuring triangular fuzzy numbers. Its suitability for a broader spectrum of fuzzy optimization problems, incorporating diverse fuzzy number types or structures, may be constrained. The paper should delve into the scope and potential restrictions of the proposed approach.

Dependency on robust ranking

The efficacy of the methodology relies heavily on the robust ranking method to convert fuzzy numbers into crisp values. The suitability of this method may vary based on the characteristics of the fuzzy numbers in question. The paper should provide insights into situations where robust ranking might encounter challenges or might not be the most appropriate technique.

Sensitivity to problem formulation

After transforming fuzzy numbers, the proposed approach formulates a crisp nonlinear programming problem. However, the sensitivity of the results to variations in the formulation or structure of the original FNPPs is not thoroughly discussed. Distinct formulations may yield different crisp representations, necessitating a more exhaustive exploration of these variations.

Lack of comparative analysis

The paper lacks a meticulous comparative analysis with alternative methodologies or existing approaches for solving FNPPs. A comparative assessment with other well-established techniques could offer a more holistic evaluation of the strengths and weaknesses of the proposed methodology.

Assumption of fuzziness in decision parameters

The paper assumes the intrinsic presence of fuzziness in decision parameters as a key characteristic of the addressed problems. However, in real-world applications, decision parameters may not consistently exhibit fuzziness. Consequently, the proposed methodology might be overly complex for scenarios where fuzziness is minimal. The paper should discuss the relevance and practicality of the proposed approach in such situations. Addressing these limitations would augment the clarity and comprehensiveness of the paper, providing readers with a more nuanced understanding of the strengths and potential constraints of the proposed methodology.

7 | Advantages

Innovative approach

The paper introduces an innovative approach that combines robust ranking methods with Kuhn-Tucker optimization conditions. This novel integration addresses the complexities associated with FNPPs and provides a fresh perspective on problem resolution.

Applicability to triangular fuzzy numbers

The methodology is specifically designed for FNPPs characterized by triangular fuzzy numbers. This specialization ensures a targeted and tailored approach to a specific class of problems, potentially leading to more effective solutions in relevant contexts.

Clarity in problem transformation

Using robust ranking methods contributes to transforming fuzzy numbers into precise and tangible values. This step enhances the clarity and applicability of subsequent analyses, making the problem formulation more straightforward and deterministic.

Systematic exploration with Kuhn-Tucker conditions

Leveraging the Kuhn-Tucker conditions for systematic exploration and analysis of the problem space adds rigor to the optimization process. This approach helps identify optimal solutions by considering both constraints and objectives, contributing to a comprehensive problem-solving methodology.

Enhanced precision and efficiency

Integrating robust ranking with Kuhn-Tucker optimization aims to improve the precision and efficiency of solving FNPPs. The proposed methodology seeks to provide more reliable and accurate solutions by addressing inherent fuzziness in decision parameters.

Demonstration through numerical examples

The paper supports its methodology with numerical examples, demonstrating the practical application of the proposed approach. These examples illustrate the methodology's effectiveness and provide a tangible understanding of its implementation.

Contributions to fuzzy optimization field

The research contributes to the broader field of fuzzy optimization by presenting a methodological pathway that integrates robust ranking and Kuhn-Tucker optimization. This contribution may advance the understanding and application of fuzzy optimization techniques in various problem domains.

Insights into optimal solutions

The systematic exploration facilitated by the Kuhn-Tucker conditions allows for identifying optimal solutions. This aspect is crucial for decision-makers seeking reliable and efficient solutions in complex and uncertain decision environments.

Versatility in handling different types of fuzzy numbers

The paper mentions the versatility of robust ranking in handling various types of fuzzy numbers, as demonstrated by its application to fuzzy Octagonal numbers. This suggests a potential adaptability of the proposed methodology to different types of fuzzy numbers, enhancing its applicability.

Integration of fuzzy set theory Milestones

The paper acknowledges and builds upon significant milestones in fuzzy set theory by referencing Zadeh et al. [3]. This integration places the proposed methodology within the broader context of established theoretical foundations in the field. In summary, the advantages of this paper lie in its innovative approach, targeted applicability to triangular fuzzy numbers, clarity in problem transformation, systematic exploration with Kuhn-Tucker conditions, and potential contributions to the broader field of fuzzy optimization. Using numerical examples further enhances the practical understanding and feasibility of the proposed methodology.

8 | Conclusion

This paper significantly contributes by addressing FNPPs characterized by triangular fuzzy numbers. The proposed methodology uses robust ranking to convert fuzzy numbers into precise values and applies the Kuhn-Tucker conditions for optimization. This integration aims to enhance the precision and efficiency in solving complex problems within this domain. The paper formulates the FNLPP and presents the Kuhn-Tucker conditions in their fuzzy form. It includes a numerical example that demonstrates the transformation

of a fuzzy problem into a crisp form using the robust ranking method, followed by solving the resulting crisp nonlinear programming problem. This methodology provides two numerical examples, and optimal solutions are obtained. The results are compared with established optimal values from previous studies, highlighting the effectiveness of this approach. Ultimately, this research offers a systematic and robust methodology for tackling FNPPs involving triangular fuzzy numbers. The integration of robust ranking and Kuhn-Tucker optimization provides a comprehensive solution for addressing the complexities introduced by fuzziness in decision parameters, with numerical examples illustrating the practical application and efficacy of the proposed methodology in achieving optimal solutions.

9 | Future Scope

Future research in fuzzy nonlinear programming can explore several promising directions. Extending the methodology to other types of fuzzy numbers, such as trapezoidal or Gaussian, can broaden its applicability. Integrating machine learning techniques may offer adaptive, data-driven optimization solutions, especially for complex datasets. Hybrid approaches combining the proposed ranking methods with metaheuristic algorithms, like genetic algorithms or PSO, could improve search capabilities and convergence speed. Applying the methodology to real-world problems in finance, supply chain management, and engineering would demonstrate its practical effectiveness. Another potential direction is adapting the approach for dynamic fuzzy environments, where decision parameters evolve over time. Developing user-friendly tools to automate ranking and optimization processes would enhance accessibility for practitioners. Conducting rigorous comparative studies with state-of-the-art methods would validate the approach.

Additionally, extending the methodology to multiobjective fuzzy programming problems, performing sensitivity analyses to assess robustness, and exploring its applicability under different optimization conditions would further enhance its versatility and reliability. This paper suggests that future works should integrate new ranking functions into various frameworks, develop efficient algorithms for large-scale problems, explore separable FNLP problems, enhance computational efficiency, and address challenges related to fuzzy parameters. These directions aim to advance the field of fuzzy optimization further and improve its practical applications.

Author Contribution

Conceptualization: Bharathi Dharmaraj; methodology: Bharathi Dharmaraj, Saraswathi Appasamy, and S.A Edalatpanah; validation: Saraswathi Appasamy and S.A Edalatpanah; formal analysis: Bharathi Dharmaraj and Saraswathi Appasamy; investigation: Saraswathi Appasamy and S.A.Edalatpanah; data maintenance: Bharathi Dharmaraj; writing-creating the initial design: Bharathi Dharmaraj; writing reviewing and editing: Bharathi Dharmaraj; visualization: Saraswathi Appasamy and S.A. Edalatpanah; monitoring: Saraswathi Appasamy; funding procurement: S.A. Edalatpanah. All authors have read and agreed to the published version of the manuscript.

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Data Availability

The dataset utilized in this research can be obtained by contacting the corresponding author and considering any ethical or legal constraints that may apply.

Conflicts of Interest

The authors declare no conflict of interest.

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